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Charge-Field Formulation of Quantum Electrodynamics (QEMED)

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By expressing classical electron theory in terms of "charge-field" functional structures, it is shown that a finite formulation of the classical electrodynamics of point charges emerges in a simple and elegant fashion. The classical charge-field form of microscopic electron theory plays the role of a covering theory for "renormalized classical electron theory," with the distinct advantage that this is accomplished by a *dynamic* subtraction mechanism, built into the theory. We then generalize this formalism into a hole-theoretic, second-quantized Dirac formulation, in order to construct a "charge-field" quantum electrodynamic theory, and discuss its basic properties. We find, in addition to the possibility that the finiteness of the classical theory may be propagated into the quantum field theory, that interacting photon states are generated as a secondary manifestation of electron–positron quantization, and do not require the usual "free" canonical quantization scheme. We discuss the possibility that this approach may lead to a better formulation of quantum electrodynamics in the Heisenberg picture and suggest a crucial experimental test to distinguish this new charge-field quantum electrodynamics "QEMED" from the standard QED formulation. Specifically QEMED predicts that the "Einstein principle of separability" should be found to be valid for correlated photon polarization measurements, in which the polarizers are changed more rapidly than a characteristic photon travel time. Such an experiment (Aspect, 1976) can distinguish between QEMED and QED in a complete and clear-cut fashion.

> *Science, like art, admits aesthetic criteria; it seeks theories that display proper conformity of the parts to one another, and to the whole, while still showing some strangeness in their proportion.*
>
> Chandrasekhar (1979).

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1. INTRODUCTION

The "elementary measurement" field-theoretical approach to electrodynamic processes in the microcosm uses the new field-theoretical language of "charge fields." This charge-field approach to electrodynamic processes is based on the paradigm that charges and their associated electromagnetic fields are permanently connected in elementary charge-field functional structures, with physical processes being described by the mutual elementary measurement interactions between various charge-field entities in the system. The classical elementary measurement electrodynamics (CEMED) theory (Leiter, 1969, 1974, 1980), is a completely finite and self-consistent electromagnetic theory, which contains an *operational*, *dynamic*, subtraction mechanism (built into the formalism) which guarantees finite observables. We will develop a second-quantized version of the charge-field formalism, in this paper, by first formulating the classical theory in its most natural form, which requires the Maxwell charge field to be an $N \otimes N$ matrix, for a theory of $N \geqslant 2$ classical electrons in interaction. The *absence* of the coulombic self-energy infinity is shown to be *enforced* by the *dynamic structure* of the charge-field interaction occurring in the Lagrangian. The matrix formalism is then generalized into a Dirac hole-theoretic, second-quantized version via the intermediary step of semiclassical quantization, upon which Dirac anticommutation relations are then imposed. In this charge-field formulation of quantum electrodynamics, "photons" are spontaneously generated via the electron–positron quantization, and no separate quantization of the electromagnetic field is required. We will call this charge-field theory Quantum Elementary Measurement Electrodynamics, QEMED.

2. A BRIEF REVIEW OF THE CLASSICAL ELECTRODYNAMICS OF CHARGE FIELDS

The initial formal structure of the classical electrodynamics of charge fields (CEMED) was first developed by Leiter (1969, 1974). However, a more elegant formulation (an $N \otimes N$ matrix formulation presented here) will allow us to proceed with the generalization into second quantization. To see this let us now consider "N" point charges ($N \geqslant 2$), whose trajectory dynamic variables are represented in the matrix form (Leiter, 1980),

$$x_\mu = \begin{bmatrix} x_\mu^{(1)} & & & \\ & x_\mu^{(2)} & & 0 \\ & & \ddots & \\ & 0 & & x_\mu^{(N)} \end{bmatrix} \tag{2.1}$$

Associated with this matrix will be a current density matrix

$$J_\mu=\begin{bmatrix} J_\mu^{(1)} & & & \\ & J_\mu^{(2)} & & 0 \\ & & \ddots & \\ & 0 & & J_\mu^{(N)} \end{bmatrix}, \qquad J_\mu^{(K)}=\frac{q(K)}{c}\frac{dx_\mu^{(K)}}{dt}\delta^3(\mathbf{x}-\mathbf{x}^{(K)}(t)) \tag{2.2}$$

a mass matrix

$$M=\begin{bmatrix} m(1) & & & \\ & m(2) & & 0 \\ & & \ddots & \\ & 0 & & m(N) \end{bmatrix} \tag{2.3}$$

a proper-time matrix and a 4-velocity matrix given by u^μ, where

$$d\tau=\gamma^{-1}\,dt, \qquad \gamma=\begin{bmatrix} \gamma(1) & & & \\ & \gamma(2) & 0 & \\ & & \ddots & \\ & 0 & & \gamma(N) \end{bmatrix},$$

$$\gamma(K)=\frac{1}{\left\{1-\left[v^{(K)}\right]^2/c^2\right\}^{1/2}} \tag{2.4}$$

$$u^\mu=d\tau^{-1}(dx^\mu)=\gamma(dx^\mu/dt) \tag{2.5}$$

and finally the "N" Maxwell charge-field degrees of freedom, represented in terms of the matrix form, $N\geqslant 2$,

$$A_\mu=\begin{bmatrix} A_\mu^{(1)} & & & \\ & A_\mu^{(2)} & & 0 \\ & & \ddots & \\ & 0 & & A_\mu^{(N)} \end{bmatrix} \tag{2.6}$$

and

$$F_{\mu v}=(A_{\mu,v}-A_{v,\mu}), \qquad \tilde{F}_{\mu v}=\tfrac{1}{2}\epsilon_{\mu v\alpha\beta}F^{\alpha\beta} \tag{2.7}$$

The action integral which gives the finite formulation of the charge-field electrodynamics in this matrix language is (note: Tr means matrix trace)

$$I=\int \operatorname{Tr}\left(Mc^2\sqrt{u_\mu u^\mu}\;dT\right)+\int dx^4\left[\operatorname{Tr}(F_{\mu v})\operatorname{Tr}(F^{\mu v})-\operatorname{Tr}(F_{\mu v}F^{\mu v})\right.$$
$$\left.+\operatorname{Tr}(J_\mu)\operatorname{Tr}(A^\mu)-\operatorname{Tr}(J_\mu A^\mu)\right]c \tag{2.8}$$

It is straightforward to see that the interaction terms $J_\mu^{(K)}A^{\mu(K)}$, $K=1,2,\ldots,N$, which will be associated with the time-symmetric self-Coulomb interaction, are dynamically excluded from this action, and hence never reappear to create self-energy infinites in the equations of motion. We also note that, before the variation, the point charge current matrix in equation (2.2) obeys the current conservation identity

$$J_\mu{}^{,\mu}\equiv 0 \tag{2.9}$$

To obtain the equations of motion from (2.8) we vary the matrix dynamical variables $\delta_x^\mu, \delta A^\mu$, respectively, to obtain the matrix equations[1]

$$M(du^\mu/d\tau)=\int dx^3\{\operatorname{Tr}(F_v{}^\mu)-F_v{}^\mu\}J^v \tag{2.10}$$

and

$$(\operatorname{Tr}(F_{\mu v})-F_{\mu v})^{,v}=(\operatorname{Tr}(J_\mu)-J_\mu) \tag{2.11}$$

Taking the matrix trace of equation (2.11) yields

$$(N-1)\operatorname{Tr}(F_{\mu v}{}^{,v})=(N-1)\operatorname{Tr}(J_\mu) \tag{2.12}$$

now since $N\geqslant 2$ in order for the charge-field formalism to contain interactions, then $(N-1)\neq 0$ and (2.11) and (2.12) yield

$$F_{\mu v}{}^{,v}=J_\mu, \qquad \bar{F}_{\mu v}{}^{,v}\equiv 0 \tag{2.13}$$

Although this equation looks formally like the usual form of Maxwell's equations, it is actually an $N\otimes N$ matrix equation ($N\geqslant 2$), associated with the particle equation of motion (2.10). It is specifically this ($N\geqslant 2$)-dimensional matrix property (associated with the charge-field concept) that each charged particle has a particular Maxwell charge-field dynamical variable structure, before the variation is performed, which allows (2.8) and (2.10) to yield a finite formulation of classical electron theory, without infinities occurring, since if (2.6) is chosen to be a matrix "scalar" ($N=1$),

[1]We will use the notation $F_{\mu v}{}^{,v}\equiv\delta^v F_{\mu v}$

then (2.8) and (2.10) contain no electromagnetic interaction, and are empty. The electromagnetic energy-momentum tensor, from (2.10) and (2.13), is of the form

$$T_{\mu v}=\eta_{\mu v}\frac{\mathrm{Tr}(F_{\alpha\beta})\mathrm{Tr}(F^{\alpha\beta})-\mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta})}{4}+\mathrm{Tr}(F_{\mu\alpha})\mathrm{Tr}(F_v^{\alpha})-\mathrm{Tr}(F_{\mu\alpha}F_v^{\alpha}) \tag{2.14}$$

and it obeys the conservation law

$$T_{\mu v}{}^{,v}=\mathrm{Tr}(J_v)\mathrm{Tr}(F_\mu{}^v)-\mathrm{Tr}(J_vF_\mu{}^v) \tag{2.15}$$

From (2.15) the "electromagnetic" energy associated with this formalism is

$$\mathcal{E}_{(\mathrm{em})}=\int dx^3\,T_{00}=\int dx^3\left[\frac{(\mathrm{Tr}(\mathbf{E}))^2+(\mathrm{Tr}(\mathbf{B}))^2-\mathrm{Tr}((\mathbf{E})^2+(\mathbf{B})^2)}{2}\right] \tag{2.16}$$

which is *not* positive definite, for *like* charges, necessarily.

However we can insure that (2.16) *is* positive definite, for *like* charges, by properly imposing the necessary boundary conditions on (2.13). The most general "charge-field" solution to (2.13) is a mixture of time-symmetric and time-antisymmetric charge-field potentials $a_{\mu_{(\pm)}}(x)$ where, in the Lorentz gauge, $a_{\mu_{(\pm)}}{}^{,\mu}=0$ and $D_{\pm}(x)\equiv[D(x)_{\mathrm{ret}}\pm D_{\mathrm{adv}}(x)]/2$

$$a_{\mu_{(\pm)}}(x)\equiv\int dx'^4 D\pm(x-x')J_\mu(x') \tag{2.17}$$

A simple formal solution to (2.13) which involves only charge fields (2.17), the homogeneous solutions being set to zero by proper boundary conditions, is

$$A_\mu=a_{\mu_{(+)}}+\mathrm{Tr}(\lambda a_{\mu_{(-)}}) \tag{2.18}$$

where λ is a matrix which determines the degree of mixing of the time-symmetric and time-antisymmetric charge fields occurring in (2.16). It can easily be seen that the proper boundary condition which makes the electromagnetic energy positive definite for like charges, while choosing a retarded time arrow on the mutual electromagnetic fields, requires that

$$\lambda=(I)/(N-1),\qquad N\geqslant 2 \tag{2.19}$$

For this choice of boundary condition on the charge-field solutions to (2.13), the solutions can be written as

$$A_{\mu} = a_{\mu_{(+)}} + \mathrm{Tr}(a_{\mu_{(-)}})/(N-1) \tag{2.20}$$

and, when inserted into (2.16), yields

$$\mathcal{E}_{(\mathrm{em})} = \mathcal{E}_{(\mathrm{ret})} + \mathcal{E}_{(\mathrm{TCRF})} \tag{2.21}$$

where

$$\mathcal{E}_{(\mathrm{ret})} \equiv \tfrac{1}{2}\int dx^3\left[(\mathrm{Tr}(\mathbf{e}_{\mathrm{ret}}))^2 + (\mathrm{Tr}(\mathbf{b}_{\mathrm{ret}}))^2 - \mathrm{Tr}\left((\mathbf{e}_{(+)})^2 + (\mathbf{b}_{(+)})^2\right)\right] \tag{2.22}$$

is the already finite reminder of the "automatically renormalized," retarded electromagnetic energy (occurring automatically via the dynamics of the formalism) and the "total coupled radiation field" $\mathcal{E}_{(\mathrm{TCRF})}$ is

$$\mathcal{E}_{(\mathrm{TCRF})} \equiv \int dx^3 \frac{(\mathrm{Tr}(\mathbf{e}_{(-)}))^2 + (\mathrm{Tr}(\mathbf{b}_{(-)}))^2}{2(N-1)} \tag{2.23}$$

For like charges, both (2.23) and (2.22) are positive, thus making (2.21) positive definite as required. In these equations, $\mathbf{e}_{(\pm,\mathrm{ret})}$ and $\mathbf{b}_{(\pm,\mathrm{ret})}$ are calculated from $F_{\mu\upsilon(\pm,\mathrm{ret})} = (a_{\mu,\upsilon} - a_{\upsilon,\mu})_{(\pm,\mathrm{ret})}$. Note that $\mathrm{Tr}(F_{\mu\upsilon_{(-)}})$, the "total coupled radiation field" of the system, takes over the physical role played by the "free" radiation field of conventional Maxwell–Lorentz theory. Now having chosen boundary conditions on the solutions on (2.13) so that for like charges, equation (2.21) is positive definite, leading to equations of the form of (2.20), we substitute (2.20) into the equations of motion for charged particles (2.10) to obtain

$$\left(\mathrm{Tr}(F_{\mu\upsilon}) - F_{\mu\upsilon}\right) = \mathrm{Tr}(F_{\mu\upsilon_{(\mathrm{ret})}}) - F_{\mu\upsilon_{(\mathrm{ret})}} + F_{\mu\upsilon_{(-)}} \tag{2.24}$$

and hence the particle equations of motion become

$$M\frac{du^{\mu}}{dt} = \int dx^3\left[\left(\mathrm{Tr}(F_{\upsilon_{(\mathrm{ret})}}{}^{\mu}) - F_{\upsilon_{(\mathrm{ret})}}{}^{\mu} + F_{\upsilon_{(-)}}{}^{\mu}\right)J^{\upsilon}_{(x)}\right] \tag{2.25}$$

This is directly seen to be equivalent to

$$m(K)\frac{du^{\mu(K)}}{d\tau^{(K)}} = \frac{q(K)u^{\upsilon(K)}}{c}\left[\sum_{\substack{J\neq K\\=1}}^{N}\left(F_{\upsilon}^{(J)\mu}{}_{(\mathrm{ret})}\right) + F_{\upsilon}^{(K)\mu}{}_{(-)}\right] \tag{2.26}$$

which is the finite, physical Lorentz–Dirac equation for classical electrons. This has been obtained by a dynamic, operational subtraction mechanism, built into the theory, and resolves in an elegant fashion the problem of the infinites in microscopic classical electron theory. It has also been shown that a macroscopically averaged charge-field of the form (Clifford, 1975)

$$F_{\mu\upsilon(\text{macro})} \equiv \langle \operatorname{Tr}(F_{\mu\upsilon}) \rangle_{(\text{sp.av.})} = \left\langle \left(\frac{1}{N-1} \right) \sum_{K=1}^{N} \left[\left[\sum_{\substack{J \neq K \\ =1}}^{N} F^{(J)}_{\mu\upsilon\,(\text{ret})} \right] + F^{(K)}_{\mu\upsilon\,(-)} \right] \right\rangle_{(\text{sp.av.})} \tag{2.27}$$

(where sp.av. ≡ space average) is a field which can be associated with the usual macroscopic Maxwell–Lorentz electrodynamics, as an approximation in the macrocosm. However, we now see that Maxwell–Lorentz electrodynamics may not be fundamental in the microcosm since it can be derived as an *approximation* to the microscopic charge-field formalism in the macrocosm. This suggests that the inherent self-energy infinities in Maxwell–Lorentz theory, which appear in its microscopic application may not be fundamental, since they are induced by the application of an essentially macroscopic approximation to a microscopic domain. If this is true it may also be that the self-energy infinities in conventional quantum electrodynamics are not fundamental for the same reason, since QED is essentially a second-quantized version of Maxwell–Lorentz theory. The real test of this assertion is the generalization of the charge-field theory to the second-quantized domain. In the next section we present a second-quantized version of "charge-field" quantum electrodynamics, and study its properties in regard to this question.

3. THE QUANTUM ELECTRODYNAMICS OF MUTUALLY INTERACTING CHARGE FIELDS, (QUANTUM ELEMENTARY MEASUREMENT ELECTRODYNAMICS)—QEMED

The second quantization of the classical charge-field theory of electrodynamics is most easily obtained by generalizing the classical theory into a "semiclassical" form, and then imposing Dirac anticommutation relations (in the context of a "hole-theoretic" electron–positron formalism). No separate commutation relations need be imposed on the Maxwell charge fields since they will be automatically quantized via their *inseparable* functional relationship to the Dirac current operators. In this manner we will see in what follows that "charge-field photons" will be automatically induced by the fermion electron–positron quantization. Since we will also show that these dynamically generated charge-field photons have all the properties of the quanta of the radiation field (now generated by electron–positron quantization) there will be no need to reintroduce them into the charge-field quantum electrodynamics. This will generate a distinct calculational advantage in that the Hilbert space of the formalism will not require an indefinite metric in order to be expressed in a manifestly relativistic covariant form. We begin our semiclassical generalization of the formalism by simply replacing the matrix array of point charge trajectories by the N column of first-quantized Dirac wave functions

$$(\psi(x)) \equiv \begin{bmatrix} \psi(x)^{(1)} \\ \vdots \\ \psi(x)^{(N)} \end{bmatrix} \equiv \psi(x), \qquad 2 \leqslant N < \infty \tag{3.1}$$

and then noting that this implies that the matrix current j_μ now obtains nonzero off-diagonal elements, associated with exchange currents,

$$j_\mu(x) = -e \begin{bmatrix} \bar{\psi}^{(1)}(x) \\ \vdots \\ \psi^{(N)}(x) \end{bmatrix} \gamma_\mu(\psi^{(1)}(x), \ldots, \psi^{(N)}(x)) = \begin{bmatrix} j_\mu^{(11)} & \cdots & j_\mu^{(1N)} \\ \vdots & \ddots & \vdots \\ j_\mu^{(N1)} & \cdots & j_\mu^{(NN)} \end{bmatrix} \tag{3.2}$$

So, via the charge-field concept, there must be off-diagonal "exchange" charge fields in the matrix Maxwell charge-field A_μ as

$$A_\mu = \begin{bmatrix} A_\mu^{(11)} & \cdots & A_\mu^{(1N)} \\ \vdots & \ddots & \vdots \\ A_\mu^{(N1)} & \cdots & A_\mu^{(NN)} \end{bmatrix} \tag{3.3}$$

Note, however, the fact that this implies that since $(\bar{\psi}^{(K)}\gamma_\mu\psi^{(J)})^* = (\bar{\psi}^{(J)}\gamma_\mu\psi^{(K)})$, then in (3.2)

$$j_\mu^* = j_\mu^{\mathrm{T}} \tag{3.4}$$

where T means matrix transpose on K,J labels. Hence we must be careful in generalizing the classical action so as to guarantee that the Hamiltonian energy density will be Hermitian. We have found a successful quantum mechanical action, with these properties (which goes over into the previous classical action where the exchange currents and charge fields are negligible), to be, in $\hbar = c = 1$ units,

$$
\begin{aligned}
I = &\int dx^4 \frac{\bar{\psi}(-i\partial\!\!\!/ + M)\psi + \text{H.c.}}{2} \\
&+ \int dx^4 \left[\frac{\operatorname{Tr}(F_{\mu\nu})\operatorname{Tr}(F^{\mu\nu}) - \operatorname{Tr}(F_{\mu\nu}F^{\mu\nu})}{4} \right. \\
&\left. + \left(\operatorname{Tr}(J_\mu)\operatorname{Tr}(A^\mu) - \operatorname{Tr}(J_\mu A^\mu)\right) \right]
\end{aligned}
\tag{3.5}
$$

where, as in the classical case, the current J_μ obeys a current conservation identity before the variation. This can only be obtained, in the quantum mechanical case, by introducing in (3.5) the covariant *nonlocal* "transverse" 4-current defined as

$$J_\mu(x) \equiv \int dx^4 P_\mu{}^\alpha(x - x') j_\alpha(x') \tag{3.6}$$

where $P_\mu{}^\alpha(x - x')$ is defined as

$$P_\mu{}^\alpha(x - x') \equiv \left(\delta_\mu{}^\alpha \delta^4(x - x') - \partial_\mu \partial^\alpha D_{(+)}(x - x')\right) \tag{3.7}$$

It is straightforward to check that $J_\mu{}^{,\mu} \equiv 0$ is valid before and after the variation of (3.5). Thus we have a complete analogy to the classical charge-field formalism, and are ready to carry out the variation of $\delta\psi, \delta\bar{\psi}$, and δA_μ to obtain the "first-quantized" Dirac and Maxwell equations of motion, in this new charge-field context, as

$$\left[-i\partial\!\!\!/ + M - e\gamma^\mu(\operatorname{Tr} a_\mu - a_\mu{}^*)\right]\psi = 0 \tag{3.8}$$

where $\gamma^\mu \equiv \gamma^\mu \otimes I_{N \otimes N}$, and

$$\left(\operatorname{Tr}(F_{\mu\nu}) - F_{\mu\nu}{}^*\right)^{,\nu} = \left(\operatorname{Tr}(J_\mu) - J_\mu{}^*\right) \tag{3.9a}$$

with

$$F_{\mu\nu}{}^* = F_{\mu\nu}{}^T, \qquad J_\mu{}^* = J_\mu{}^T \tag{3.9b}$$

Note, in (3.8), the charge-field a_μ is related to A_μ by

$$a_\mu(x) \equiv \int dx'^4 P_\mu{}^\alpha(x - x') A_\alpha(x') \tag{3.10a}$$

$$F_{\mu\nu} = \left(A_{\mu,\nu} - A_{\nu,\mu}\right) = \left(a_{\mu,\nu} - a_{\nu,\mu}\right) \tag{3.10b}$$

It is the presence of the transverse 4-current (3.6) in the action (3.5) which causes a_μ to appear in (3.8), and also guarantees that conservation of current $J_\mu{}^{,\mu}=0$ is satisfied, as is required by the charge-field Maxwell equations (3.9). In cases where the exchange currents are negligible (i.e., for physical situations in which wave-function overlap is small between $\psi^{(K)}$ and $\psi^{(J)}$) the transverse 4-current will go over into the usual "direct" current $J_\mu^{(KK)} \to j_\mu^{(KK)}$, which will automatically be conserved by the equations of motion. In cases where the exchange currents are important, it is the full transverse 4-current (3.6) which is conserved, as is implied by the presence of a_μ and *not* A_μ in the Dirac equations of motion. At this point it is instructive to clarify the role of gauge transformations in this formalism. First of all we note that the charge field a_μ obeys gauge condition identically

$$a_\mu{}^{,\mu} \equiv 0 \tag{3.11}$$

while the charge-field A_μ has an *arbitrary* value of $A_\mu{}^{,\mu}$. However it is easy to verify that arbitrary gauge transformations on A^μ leave (3.11) unchanged. The reason why the Lorentz-gauged charge field a_μ appears in (3.8), instead of A_μ, is because of the presence of the transverse 4-current (3.6) in the action (3.5). For example, we see that in the interaction part of the Langrangian

$$\int dx^4 \operatorname{Tr}(J_\mu)\operatorname{Tr}(A_\mu) = \int dx^4 \operatorname{Tr}(j_\mu)\operatorname{Tr}(a^\mu) \tag{3.12a}$$

$$\int dx^4 \operatorname{Tr}(J_\mu A^\mu) = \int dx^4 \operatorname{Tr}(j_\mu a^\mu) \tag{3.12b}$$

so that when we vary $\delta\bar{\psi}$, it is actually a_μ and *not* A_μ which appears in the equations of motion for ψ given in (3.8). Hence the formalism is restricted to have only the Lorentz-gauge freedom, because of the use of the transverse 4-current, required by current conservation. However, even though the physically observed charge field is a_μ (as seen by ψ in its equation of motion), we can still make gauge transformations on a_μ itself. This might be like

$$a'_\mu = (a_\mu + \partial_\mu \Lambda) \tag{3.13}$$

where $\Box\Lambda = 0$. In the charge-field context, this would imply that Λ would be of the general charge-field form

$$\Lambda(x) \equiv \left[\int dx'^4 D_{(-)}(x-x') S(x') \right] \otimes I \tag{3.14}$$

where $S(x')$ could be a real "source" function of ψ, e.g., $S=(\bar{\psi}\psi)$. Then since this implies that, for $N \geqslant 2$,

$$(\mathrm{Tr}(a'_\mu)-(a'_\mu)^*)=\left[(N-1)\partial_\mu\Lambda+(\mathrm{Tr}(a_\mu)-a_\mu{}^*)\right] \tag{3.15}$$

we see that (3.8) will remain form invariant if we simultaneously make a gauge transformation on ψ as

$$\psi'(x)=\exp\left[-i(N-1)\lambda(x)\right]\psi(x) \tag{3.16}$$

where $\lambda(x)=\int dx'^4 D(x-x')S(x')$, then

$$\begin{aligned}\left[-i\partial\!\!\!/+M-e\gamma^\mu(\mathrm{Tr}(a'_\mu)-a'^*_\mu)\right]\psi' &= \left[-i\partial\!\!\!/+M-e\gamma^\mu(\mathrm{Tr}(a_\mu)-a_\mu{}^*)\right]\psi \\ &=0\end{aligned} \tag{3.17}$$

is valid. Hence this formalism is a gauge theory for charge-field gauge freedom restricted to the Lorentz gauge group. However since electrodynamic observables will always be gauge invariant, this restriction will not produce any observable effect on the gauge-invariant quantities in the formalism. Having clarified the nature of gauge transformations, we now study the total energy of the system as obtained from the canonical energy-momentum tensor $T_{\mu v}$ associated with the action principle (3.5). It is easily obtained as

$$\begin{aligned}\mathcal{E} = \int dx^3 T_{00} = \int dx^3 \Big\{ & \left[\tfrac{1}{2}(\psi^\dagger(\boldsymbol{\alpha}\cdot\boldsymbol{\rho}+\beta M)\psi+\mathrm{H.c.})\right] \\ & -\left[\mathrm{Tr}(\mathbf{j})\cdot\mathrm{Tr}(\mathbf{a})-\mathrm{Tr}(\mathbf{j}\cdot\mathbf{a})\right] \\ & +\tfrac{1}{2}\left[(\mathrm{Tr}(\mathbf{E}))^2+(\mathrm{Tr}(\mathbf{B}))^2-\mathrm{Tr}(\mathbf{E}\cdot\mathbf{E}+\mathbf{B}\cdot\mathbf{B})\right]\Big\}\end{aligned} \tag{3.18}$$

Now in order for the theory to be physical, the total energy must be positive definite. In a manner similar to the classical case discussed in Section 2, the charge-field solution to (3.9) which in its "direct" charge-field structure is of a similar form as that of (2.19) and (2.20), $N \geqslant 2$, as

$$a_\mu=\left\{a_{\mu(+)}+\left[\mathrm{Tr}(a_{\mu(-)})/(N-1)\right]\right\} \tag{3.19}$$

except than now, since J_μ contains off-diagonal exchange currents, this means that

$$a_{\mu(\pm)}(x)\equiv\int dx'^4 D_{(\pm)}(x-x')J_\mu(x') \tag{3.20}$$

contains off-diagonal charge fields in (3.19). In particular the components of (3.19) are ($N \geqslant 2$)

$$a_\mu^{(KK)} = \left[a_\mu^{(KK)}{}_{(+)} + \left(\frac{1}{N-1} \right) \sum_{J=1}^{N} a_\mu^{(JJ)}{}_{(-)} \right] \tag{3.21a}$$

$$a_\mu^{(KJ)} = \left(a_\mu^{(KJ)}{}_{(+)} \right), \qquad (K \neq J) \tag{3.21b}$$

When substituted into the equation of motion (3.8), this gives us

$$\left[-i\partial\!\!\!/ + M - e\gamma^\mu \left(\mathrm{Tr}(a_{\mu_{(-)}}) + \left(\mathrm{Tr}(a_{\mu_{(+)}}) - a^*_{\mu\,(+)} \right) \right) \right] \psi = 0 \tag{3.22}$$

or equivalently, since $(a_{\mu_{(+)}} + a_{\mu_{(-)}}) = a_{\mu_{(\mathrm{ret})}}$, (3.22) is also of the physically transparent form

$$\left[-i\partial\!\!\!/ + M - e\gamma^\mu \left(\mathrm{Tr}(a_{\mu_{(\mathrm{ret})}}) - a^*_{\mu\,(+)} \right) \right] \psi = 0 \tag{3.23}$$

This is the "first-quantized" generalization of the classical charge-field electrodynamic formalism presented earlier; however, it still is not complete in this form, since the negative-energy Dirac solutions in (3.22), (3.23) could still make the energy (3.18) not positive definite in the associated Hilbert space. This is a reflection of an old difficulty associated with the Dirac equation, which is solved by "second-quantizing" the $\psi^{(K)}$ in the context of a Dirac hole-theoretic quantum field theory scheme, where it becomes a non-Abelian anticommuting operator. To implement this we impose the following canonical anticommutation relations on the $\psi^{(K)}$ as ($K, J = 1, 2, \ldots, N; N \geqslant 2$)

$$\begin{aligned} \left\{ \psi^{(K)\dagger}(x), \psi^{(J)}(x') \right\} \Big|^{t=t'} &= \delta^{(KJ)} \delta^3(\mathbf{x} - \mathbf{x}') \\ \left\{ \psi^{(K)}(x), \psi^{(J)}(x') \right\} \Big|^{t=t'} &= 0 \\ \left\{ \psi^{(K)\dagger}(x), \psi^{(J)\dagger}(x') \right\} \Big|^{t=t'} &= 0 \end{aligned} \tag{3.24}$$

For consistency we must also make the theory invariant under charge conjugation of the $\psi^{(K)}$. This requires the standard replacement of $\bar{\psi}^{(K)}\gamma_\mu\psi^J$ by $\frac{1}{2}[\bar{\psi}^{(K)}, \gamma_\mu\psi^{(J)}]$ in the theory, via (3.6) as

$$J_\mu^{(KJ)}(x) = \int dx'^4 P_\mu^\alpha(x - x') \left(\frac{-e}{2} \right) \left[\bar{\psi}^{(K)}_{(x')}, \gamma_\mu \psi^{(J)}(x') \right] \tag{3.25}$$

and also in the various charge-field operators in (3.20), (3.22), and (3.23).

Then the Hamiltonian operator (3.18) can be written in the charge-conjugate invariant, Hermitian form (where † denotes Hermitian conjugate) as

$$H=\int dx^3\left\{\tfrac{1}{4}\left(\left[\psi^\dagger,H_0\psi\right]+\left[(H_0\psi)^\dagger,\psi\right]\right)-(\mathrm{Tr}(\mathbf{j})\cdot\mathrm{Tr}(\mathbf{a})-\mathrm{Tr}(\mathbf{j}\cdot\mathbf{a}))\right.$$
$$\left.+\tfrac{1}{2}((\mathrm{Tr}(\mathbf{E}))^2+(\mathrm{Tr}(\mathbf{B}))^2-\mathrm{Tr}(\mathbf{E}\cdot\mathbf{E}+\mathbf{B}\cdot\mathbf{B}))\right\} \tag{3.26}$$

where $H_0\equiv(\boldsymbol{\alpha}\cdot\boldsymbol{p}+\beta M)$. In (3.26), the Hermitian property of H is guaranteed by the fact that $j_\mu^*=j_\mu^{\mathrm{T}}$ and $a_\mu^*=a_\mu^{\mathrm{T}}$ are true. In this second-quantized theory, the operator equation of motion for ψ, in the Heisenberg picture, is then formally similar to (3.22) as

$$\left[-i\partial\!\!\!/+M-e\gamma^\mu\left(\mathrm{Tr}(a_{\mu(-)})+\left(\mathrm{Tr}(a_{\mu(+)})-a^\dagger_{\mu(+)}\right)\right)\right]\psi=0 \tag{3.27}$$

and this must also be equivalent to

$$-i\partial_t\psi=[H,\psi] \tag{3.28}$$

since H is the operator generator of time displacement in this "charge-field" formulation of quantum electrodynamics QEMED. The consistency of this requirement with the anticommutation relations (3.24) implies additional sets of "dynamically" determined equal-time commutation relations such as

$$[a_\mu(x),\psi(x')]|^{t=t'}=0 \tag{3.29}$$

$$\left\{\tfrac{1}{2}\left[(\mathrm{Tr}(\mathbf{E}(x)))^2+(\mathrm{Tr}(\mathbf{B}(x)))^2-\mathrm{Tr}(\mathbf{E}(x)\cdot\mathbf{E}(x)+\mathbf{B}(x)\cdot\mathbf{B}(x))\right.\right.$$
$$\left.\left.-\mathrm{Tr}(\rho(x))\mathrm{Tr}(a_0(x))+\mathrm{Tr}(\rho(x)a_0(x))\right],\psi(x')\right\}|^{t=t'}=0 \tag{3.30}$$

and their corresponding equivalents for $\bar{\psi}(x')$ and $a_\mu^\dagger(x')$, where $\mathbf{E}$ and $\mathbf{B}$ are calculated from $F_{\mu v}=(a_{\mu,v}-a_{v,\mu})$, and a_μ is given by (3.19). These "dynamically induced" equal-time commutation relations represent nonlocal constraint equations on the charge fields, which are not equivalent to the use of "canonical" equal-time commutation relations on the a_μ. Hence we avoid, in QEMED, all of the pathologies associated with the use of canonical commutation relations on the a_μ, such as the need of an indefinite metric in the Hilbert space of states. This is a distinct formal advantage of QEMED over QED, and suggests that the need for an indefinite metric, in the Hilbert space, is associated with the nonoperational concept of quantizing the "free, uncoupled radiation field" with canonical commutation relations.

Now if (3.19) is inserted into (3.26), H takes on an operator form which reveals its positive definite character more explicitly as

$$H=\left\{H\begin{pmatrix}\text{interacting}\\ \text{Dirac energy}\end{pmatrix}+H\begin{pmatrix}\text{total coupled}\\ \text{radiation field}\\ \text{energy}\end{pmatrix}\right.$$

$$\left.+\left[H\begin{pmatrix}\text{retarded}\\ \text{electromagnetic}\\ \text{energy}\end{pmatrix}-H\begin{bmatrix}\text{time-symmetric}\\ \text{exchange electro}\\ \text{magnetic energy}\end{bmatrix}\right]\right\} \tag{3.31}$$

where

$$H\begin{pmatrix}\text{interacting}\\ \text{Dirac energy}\end{pmatrix}\equiv\int dx^3\left[\left(\tfrac{1}{4}\left[\psi^\dagger,H_0\psi\right]+\text{H.c.}\right)-\text{Tr}(\mathbf{j}\ )\cdot\text{Tr}(\mathbf{a})+\text{Tr}(\mathbf{j}\cdot\mathbf{a})\right] \tag{3.32}$$

$$H\begin{pmatrix}\text{total coupled}\\ \text{radiation field}\\ \text{energy}\end{pmatrix}\equiv\int dx^3\,\frac{(\text{Tr}(\mathbf{e}_{(-)}))^2+(\text{Tr}(\mathbf{b}_{(-)}))^2}{2(N-1)} \tag{3.33}$$

($N>2$), and

$$H\begin{pmatrix}\text{finite electro}\\ \text{magnetic energy}\end{pmatrix}\equiv\left[H\begin{pmatrix}\text{retarded}\\ \text{electromagnetic}\\ \text{energy}\end{pmatrix}-H\begin{bmatrix}\text{time-symmetric}\\ \text{exchange electro}\\ \text{magnetic energy}\end{bmatrix}\right]$$

$$\equiv\int dx^3\left[\frac{(\text{Tr}(\mathbf{e}_{\text{ret}}))^2+(\text{Tr}(\mathbf{b}_{\text{ret}}))^2}{2}\right.$$

$$\left.\frac{-\text{Tr}\left(\mathbf{e}_{(+)}\cdot\mathbf{e}_{(+)}+\mathbf{b}_{(+)}\cdot\mathbf{b}_{(+)}\right)}{2}\right] \tag{3.34}$$

In (3.31), the first term (3.32) has positive definite expectation values in the Hilbert space via the hole-theoretic, charge-conjugation invariant second-quantization scheme associated with using canonical commutation relations on the ψ (we call this "elementary fermion quantization" EFQ); the second term (3.33) is the sum of squares of charge-field operators and hence is positive definite automatically; the third term (3.34) is the difference of two positive definite operator terms, which while being positive definite only for states containing like charges, will always be dominated by the positive definite mass energies associated with the first term (3.32). Hence the choice of (3.19), as the charge-field solution to the Maxwell charge-field operator equations, the Hamiltonian operator (3.26), (3.31) is a positive definite operator. This will guarantee that a state with lowest energy, i.e., the vacuum state $|0\rangle$, will exist in the QEMED formalism. It is important to note that in the first and third terms in (3.31), the presence of the exchange charge-field operators generates diagonal terms which cancel out the time-symmetric self-interaction operators in the direct Maxwell charge-field terms, in an operator context. This is consistent with the presence of a similiar cancellation in the operator equations of motion (3.27), and suggests that this will lead to states in the Hilbert space with finite "self-renormalized" observables occurring automatically, owing to the operator dynamic way in which the time-symmetric self-interaction operators are dynamically excluded via the basic structure of the QEMED formalism based on the "elementarity" of the microscopic measurement process at the charge-field operator level.

As indicated earlier, we have not used "canonical commutation relations" on the photon charge-field operators, rather we invoke them in the EFQ context, only for the fermions. So one immediately asks, "where do photons enter the QEMED theory"? The answer comes from the fact that the physical effects of charge-field photons are present naturally in the form of the operator $\mathrm{Tr}(a_{\mu_{(-)}})$ which appears automatically in the operator equation of motion (3.27). To show this more explicitly let us denote this operator by the symbol $A_\mu^{(\mathrm{TCRF})}$, which we will call the "total coupled radiation field" operator of the system, as

$$\begin{aligned} A_\mu^{(\mathrm{TCRF})}(x) &\equiv \mathrm{Tr}(a_{\mu_{(-)}}) = \int dx'^4 D_{(-)}(x-x')\mathrm{Tr}(J_\mu(x')) \\ &= \left(A_\mu^{(\mathrm{TCRF})}(x)\right)^\dagger \end{aligned} \tag{3.35}$$

Now we note that $A_\mu^{(\mathrm{TCRF})}$ obeys the "photonlike" equations of motion, *as identities*,

$$\Box A_\mu^{(\mathrm{TCRF})} \equiv 0, \qquad \left(A_\mu^{(\mathrm{TCRF})}\right)^{,\mu} \equiv 0 \tag{3.36}$$

even though it is clear from its definition (3.35), that $A_\mu^{(\mathrm{TCRF})}$ is not a "free" radiation field, since it is coupled to the matrix trace of the current operator J_μ. Also from (3.28) we see that

$$-i\partial_t J_\mu = [H, J_\mu] \tag{3.37}$$

will automatically imply

$$-i\partial_t A_\mu^{(\mathrm{TCRF})} = \left[H, A_\mu^{(\mathrm{TCRF})} \right] \tag{3.38}$$

without the need of "canonical" rules on $A_\mu^{(\mathrm{TCRF})}$. This is the key advantage of the QEMED quantum electrodynamics in that it eliminates an unnecessary quantum algorithm from the physical arena, and replaces it by a more compact and conceptually beautiful EFQ approach. Operator consistency of (3.36) with (3.38) requires that we have $[A_\mu^{(\mathrm{TCRF})}(x), A_\mu^{(\mathrm{TCRF})}(x')]|^{t=t'} = 0$ being true, which is a charge-field form of "microscopic causality" at the operator level. To see how we generate photon states in the theory, via (3.35), one substitutes the Fourier transform of the $D_{(-)}(x-x')$ function into $A_\mu^{(\mathrm{TCRF})}$ and obtains an automatic decomposition of it into its positive and negative frequency parts as

$$\begin{aligned} A_\mu^{(\mathrm{TCRF})}(x) &= \int \frac{d\lambda^3}{(2\pi)^3 w(\lambda)} \left[A_\mu^{(\mathrm{TCRF})^{(+)}}(\lambda) \cdot \exp(i\lambda_v x^v) \right. \\ &\qquad \left. + A_\mu^{(\mathrm{TCRF})^{(-)}}(\lambda) \cdot \exp(-i\lambda_v x^v) \right] \\ &= \left[A_\mu^{(\mathrm{TCRF})^{(+)}}(x) + A_\mu^{(\mathrm{TCRF})^{(-)}}(x) \right] \end{aligned} \tag{3.39}$$

where

$$w(\lambda) = \sqrt{\boldsymbol{\lambda}\cdot\boldsymbol{\lambda}} = \lambda^0, \qquad \lambda_\mu \lambda^\mu = 0, \qquad \lambda^\mu A_\mu^{(\mathrm{TCRF})^{(\pm)}} = 0$$

and

$$A_\mu^{(\mathrm{TCRF})^{(\pm)}}(x) = \int \frac{d\lambda^3}{(2\pi)^3 w(\lambda)} A_\mu^{(\mathrm{TCRF})^{(\pm)}}(\lambda) \exp(\pm i\lambda_v x^v) \tag{3.40}$$

where

$$A_\mu^{(\mathrm{TCRF})^{(\pm)}}(\lambda) \equiv \left[\pm \left(\frac{i}{4} \right) \int dx'^4 \mathrm{Tr}(J_\mu(x')) \exp(\mp i\lambda_\alpha x'^\alpha) \right. \tag{3.41}$$

Now since $A_\mu^{(\mathrm{TCRF})^{(\pm)}}(x)$ is a linear functional of the operator $\mathrm{Tr}(J_\mu)$, we have from (3.37) and (3.38), after an integration by parts, that

$$\pm w(\lambda) A_\mu^{(\mathrm{TCRF})^{(\pm)}}(\lambda) = \left[H, A_\mu^{(\mathrm{TCRF})^{(\pm)}}(\lambda) \right] \tag{3.42}$$

Since H is positive definite, as indicated earlier, then a vacuum state $|0\rangle$ exists in the Hilbert space of the charge-field formalism. Then applying (3.42) to the vacuum state we have, since $H|0\rangle=0$, that

$$\pm w(\lambda)A_\mu^{(\mathrm{TCRF})^{(\pm)}}(\lambda)|0\rangle=\left[H,A_\mu^{(\mathrm{TCRF})^{(\pm)}}(\lambda)\right]|0\rangle \tag{3.43}$$

$$H\left(A_\mu^{(\mathrm{TCRF})^{(\pm)}}(\lambda)|0\rangle\right)=\pm w(\lambda)\left(A_\mu^{(\mathrm{TCRF})}(\lambda)|0\rangle\right) \tag{3.44}$$

However, since $w(\lambda)>0$; $\langle H\rangle\geqslant 0$, it must be that

$$A_\mu^{(\mathrm{TCRF})^{(-)}}(\lambda)|0\rangle=0 \tag{3.45}$$

that is to say, $A_\mu^{(\mathrm{TCRF})^{(-)}}(\lambda)$ is a destruction operator for charge-field photons on the vacuum state $|0\rangle$. Similiarly (3.44) also implies that

$$A_\mu^{(\mathrm{TCRF})^{(+)}}(\lambda)|0\rangle=|1_\lambda\rangle \tag{3.46}$$

or that $A_\mu^{(\mathrm{TCRF})^{(+)}}(\lambda)$ is a creation operator for charge-field photons on the vacuum, and that $|1_\lambda\rangle$ is a one-photon state of energy $w(\lambda)$. We can also create multiphoton states in the same manner, by operating on the vacuum $|0\rangle$ with $A_\mu^{(\mathrm{TCRF})^{(+)}}(\lambda)$ over and over again. Hence QEMED contains in its Hilbert space physical states which contain all the physical properties of photons. There will be a difference, however, associated with the Einstein principle of separability and photon measurements, which will be elaborated on in the conclusions to this paper. In all other "physical" aspects these charge-field photons, via (3.36), are equivalent to the photons generated in the standard QED. This unique algorithm is occurring in the Heisenberg picture for interacting electron–positron operators in a charge-field, elementary measurement context, and suggests that photons are not elementary particles. Instead QEMED implies that photons are secondary dynamical manifestations of electron–positron quantization, via EFQ, in a charge-field context. The fact that this is occurring in the Heisenberg picture allows the full richness of its machinery to be brought into action, in obtaining solutions to QEMED. There are several approaches one might take as follows: first of all we might attempt to set up a Heisenberg "bare-state" perturbation–iteration scheme for the formalism. This is made easy by the fact that the absence of the time-symmetric self-interaction operators in the equations of motion allow "solitons" to exist in the theory, and may yield an elegant method of obtaining solutions in both a "bound state" and "S-matrix" form. The presence of "soliton" states in QEMED is a unique property associated with its structure, which is not shared by the standard QED. This will play an important part in the inherent finiteness and self-consistency of the formalism as well as allowing "soliton" calculational techniques to be applied with ease (a distinct advantage for QEMED, since QED doesn't have this ability to capitalize on this calculational domain). This will allow insights into the Hilbert space structure of QEMED to be developed in a clear and concise manner. Next, one might attempt to develop the structure of the operator Green functions of the formalism directly, as in the Lehmann spectral forms for the two-point functions. (For example, this would be particularly useful in determining the nature of the renormalization constants and their possible finiteness in the QEMED formalism.)

Finally one might attempt to obtain information directly from the Heisenberg operator equations of motion, in an approximation scheme which involved iteration *and* direct integration techniques. This way has shown recent success in solving for radiative corrections in the standard QED (Ackerhalt and Eberly, 1974). Since this last approach may be conceptually the simplest way to obtain information about the predictions of QEMED, we will formulate this last approach more specifically in this paper. In general, the QEMED quantum electrodynamic formalism is written in terms of an $N\otimes N$ matrix operator equation 3.27, in a "layered" Hilbert space consisting of N sub-Hilbert spaces interacting with each other via this operator equation of motion, where $(2\leqslant N<\infty)$.

Suppose that we wish to study the simple problem of the interaction of a quantum mechanical electron in the presence of an "external" time-independent current, like that generated by the electric and magnetic fields of macroscopic instruments in the laboratory (to a first-order approximation). In this case, we can approximate the full N-fold formalism by an $N=2$ approximate formalism, associated with the electron–positron current operator $J_\mu^{(11)}$ and the external current $J_\mu^{(22)}\equiv J_\mu^{(\mathrm{ext})}$, which is taken to be a c number in this approximation. In this case all exchange-current operator effects between $\psi^{(1)}$ and $\psi^{(2)}$ are neglected and assumed negligible. This neglect of the exchange-operator effects will be, to first order, the manner in which the interaction of a "distinct macroscopic measuring instrument" and a microscopic quantum operator will be defined in QEMED (this will be elaborated on further in the conclusions to this paper). In this context, the current has the following structure:

$$J_\mu=\begin{bmatrix} J_\mu^{(11)} & 0 \\ 0 & J_\mu^{(\mathrm{ext})}\end{bmatrix} \tag{3.47}$$

where, because of the neglect of the exchange–charge-field effects in this approximation, (3.6) simplifies to imply

$$J_\mu^{(11)}=j_\mu^{(11)}=(-e/2)\left[\bar{\psi}^{(1)},\gamma_\mu\psi^{(1)}\right] \tag{3.48a}$$

$$J_\mu^{(22)}=J_\mu^{(\mathrm{ext})}\qquad \text{(a } c \text{ number)} \tag{3.48b}$$

Then, since only $\psi^{(1)}$ is of dynamical interest in the external field approximation, (3.19) through (3.24) and (3.27) lead to the simpler operator equations of motion for $\psi^{(1)}$ as

$$\left[-i\partial\!\!\!/+m-e\gamma^{\mu}a_{\mu}^{(\text{ext})}{}_{(\text{ret})}-e\gamma^{\mu}a_{\mu}^{(11)}{}_{(-)}\right]\psi^{(1)}=0 \tag{3.49}$$

In (3.49), since $J_{\mu}^{(\text{ext})}$ is a specified, external, time-independent, c-number current, then

$$a_{\mu}^{(\text{ext})}(x)_{(\text{ret})}=\int dx'^{3}\left(\frac{J_{\mu}^{(\text{ext})}(\mathbf{x})}{4\pi|\mathbf{x}-\mathbf{x}'|}\right)\equiv a_{\mu}^{(\text{ext})}(\mathbf{x}) \tag{3.50}$$

and $a_{\mu_{(-)}}^{(11)}$ is the remainder of the total coupled radiation field operator

$$a_{\mu}^{(11)}(x)_{(-)}=\int dx'^{4}D_{(-)}(x-x')(-e/2)[\bar{\psi}_{(x')}^{(1)},\gamma_{\mu}\psi_{(x')}^{(1)}] \tag{3.51}$$

Since the only quantum mechanical dynamics being studied are those of the anticommuting operator $\psi^{(1)}$, we can rewrite (3.47) through (3.51) in a simpler form, suppressing the charge-field indices "1" and "2" as follows

$$\left[-i\partial\!\!\!/+m-e\gamma^{\mu}a_{\mu}^{(\text{ext})}(\mathbf{x})-e\gamma^{\mu}a_{\mu_{(-)}}(x)\right]\psi(x)=0 \tag{3.52a}$$

$$a_{\mu_{(\pm)}}(x)\equiv(-e/2)\int dx'4D_{(\pm)}(x-x')\left[\bar{\psi}(x'),\gamma_{\mu}\psi(x')\right] \tag{3.52b}$$

In this simpler notation, the associated Hamiltonian operator (3.26) becomes

$$H=\int dx^{3}\left\{\frac{\left[\psi^{\dagger},H_{\text{ext}}\psi\right]+\left[(H_{\text{ext}}\psi)^{\dagger},\psi\right]}{4}-(\mathbf{j}\cdot\mathbf{a}_{(-)})\right.$$
$$\left.+\tfrac{1}{2}(\mathbf{e}_{(+)}\cdot\mathbf{e}_{(-)}+\mathbf{e}_{(-)}\cdot\mathbf{e}_{(+)}+\mathbf{b}_{(+)}\cdot\mathbf{b}_{(-)}+\mathbf{b}_{(-)}\cdot\mathbf{b}_{(+)})+\left[(\mathbf{e}_{(-)})^{2}+(\mathbf{b}_{(-)})^{2}\right]\right\} \tag{3.53}$$

where $H_{\text{ext}}\equiv[\boldsymbol{\alpha}\cdot(p+e\mathbf{a}_{\text{ext}})+\beta m-e\phi_{\text{ext}}]$. Now arguments similar to those of (3.35) through (3.46) will imply that $a_{\mu_{(-)}}(x)$ acts in a manner to create and destroy charge-field photons, associated with $\psi(x)$, in this approximation. One may now directly apply Heisenberg operator equation of motion techniques, involving interation *and* integration in tandem, to (3.52) to obtain radiative corrections to the bound states associated with $\psi(x)$ in the presence of $J_{\mu}^{(\text{ext})}(\mathbf{x})$. The details of these calculations, with respect to their similarities and differences to the standard QED results will be reported on in a future work. The presence of the operator charge-field potential

$a_{\mu_{(-)}}$ in (3.52) suggests that QEMED should predict the correct radiative corrections, similiar to that of renormalized QED, because of the earlier successes obtained with this operator by Series (1969). We also note that the absence of the self-Coulomb operator $a_{\mu_{(+)}}$ from (3.52a) suggests that its associated self-energy infinity will not occur in the QEMED radiative corrections. Hence renormalization effects will occur only, owing to virtual interactions with the vacuum, which exclude self-energy effects. However, since photons are being generated by $a_{\mu_{(-)}}$, without the use of canonical commutation relations for photons, this suggests that the virtual renormalization effects will generate finite renormalization constants. This is because the presence of infinite renormalization constants, in QED, can be traced to the presence of self-interaction, as well as the canonical quantization of free uncoupled radiation fields with canonical quantization rules. An important test of these new concepts will be in the arena of specific numerical predictions (e.g., radiative corrections to hydrogen atoms, the anomalous magnetic moment of the electron, radiative corrections to multielectron atoms, lifetime calculations to positronium, etc). We intend the basic formalism presented here to serve as a first step towards the formulation and solution to these practical tests for charge-field quantum electrodynamics QEMED. We suggest that, since QEMED is a gauge theory with $U(1)$ symmetry, then the inherent self-consistency of QEMED may make it a very attractive candidate to use as a model for gauge-theoretical formulations of the "electro-weak," and "strong interactions" (should its numerical predictions be shown to be satisfactory in the quantum electrodynamic domain). This is because it is possible to construct a generalized charge-field formalism for every "conventional" quantum gauge theory presently under investigation. This may have important implications for future improved formulations of "electro-weak" and "quantum chromodynamic" theories [with $SU(2)\times U(1)$ and higher gauge symmetries], which may be inherently infinite and "self-renormalizing" when written out in terms of the "elementary measurements" of the charge-field language presented here in the $U(1)$ context.

4. CONCLUSIONS: THE EINSTEIN PRINCIPLE OF SEPARABILITY AND QEMED VS. QED; A CRUCIAL EXPERIMENTAL TEST OF THE CHARGE-FIELD FORMULATION OF QUANTUM ELECTRODYNAMICS

The purpose of the discussion and development of the QEMED theory presented here was to demonstrate that the paradigm of "elementary measurement" between charge-fields could be extended, from its classical form, into a quantum electrodynamic domain. It leads to a new "quantum theory of elementary measurement" in the microcosm, in which the microscopic mutual, electromagnetic measurement process is well defined in the operator equations of motion of the theory, hence the name, quantum elementary measurement electrodynamics, QEMED. However, even though QEMED may have specific formal and operational advantages over QED, via finiteness and internal consistency, and even if future calculations show that QEMED is able to predict the same results about radiative corrections as QED, it would still be insufficient proof of QEMED's truth. This is because if QEMED and QED are "different" theories, they must "differ" in some important observable way. Both QEMED's agreement with experiments, and a key "observable difference" from QED, must be shown to really prove that QEMED is the correct way to formulate quantum processes in Nature. Hence what is needed, in addition to QEMED's agreement with QED's predictions about radiative corrections, is for QEMED to predict something observable that QED cannot predict, since if this new physical effect were to be seen in Nature, it would serve as a distinct test of the viability of QEMED over QED. Hence in the following paragraphs we will distinguish a basic key difference between QEMED and QED, and show how it may be clearly tested by an experiment, which is in the process of being done Aspect (1976).

To begin with, because of the matrix nature of the charge-field operators in the QEMED theory, and the associated fermion anticommutation relations (3.24), the Hilbert space, built up out of the eigenstates of the Hamiltonian operator (3.26), will have a countable, "layered," mutually interacting structure. This is because (3.24) generates in (3.26) "N" second-quantized sub-Hilbert spaces, which interact, via charge-field operators, in "mutual interaction," as described by the operator equation of motion (3.27). Among the possible states in the Hilbert space (which can be built up from the soliton structure of the formalism) there will exist "asymptotic states" for which the "exchange," off-diagonal, charge-field interactions, $(K \neq J, \langle J_\mu^{(KJ)} a_{(+)}^{(JK)\mu} \rangle)$ will have *vanishing amplitudes*, in this asymptotic domain. There will also exist states, in the Hilbert space, for which the exchange charge fields will *not* have vanishing amplitudes $\langle J_\mu^{(KJ)} a_{(+)}^{(JK)\mu} \rangle$ as well. This is a key difference between QEMED and QED, and is related to the basic structural operator difference between the two theories; involving an *absence* of time-symmetric self-Coulomb charge fields and presence of exchange charge-field operators, in an $N \geqslant 2$ matrix dimensional context. Let us consider the first class of "asymptotic states" for which $(K \neq J), \langle J_\mu^{(KJ)} a_{(+)}^{(JK)\mu} \rangle = 0$ is valid. For such states, "asymptotic correlations" may or may not be present, depending on how they were prepared. However, in any case these correlations are not "enforced", owing to $\langle J_\mu^{(KJ)} a_{(+)}^{(JK)\mu} \rangle = 0$, and may be broken down by interactions with other elementary measurements which may be present in the aforesaid asymptotic domain. We deduce this property by a study of the soliton solutions in QEMED which, of course, directly affect the character of the Hilbert space of QEMED.[2] Suppose we identify a "system to be observed" and a "measuring apparatus" with such a subset of such asymptotic states such that $\langle J_{\mu}^{(KJ)} a_{(+)}^{(JK)\mu} \rangle = 0$ for exchange charge-field operators involving "overlap" between the "system" and the "measuring instrument." (There may still be nonzero "internal" exchange interactions inside the "system" and the "measuring apparatus," but these will affect only "internal" correlations in their structure, and not affect the asymptotic "external" correlations we are considering here.) Now the very fact that such an operator-state distinction can be made, in QEMED, for a "system" and a "measuring instrument" means that this (with the inherent microscopic operator causality of QEMED, which implies that the associated operator Green's functions propagate virtual information only inside their light cones) will automatically imply that the Einstein Principle of Separability will be observable in Nature [see the recent analysis and theorems of Selleri (1979) and Bell (1976)]. This test is further clarified by the fact that formal nature of QED precludes an operator distinction between system and measuring instrument and for this reason implies that the Einstein Principle of Separability cannot be true in QED (Costa de Beauregarde, 1977). It has been proposed (Aspect, 1976) to test the Einstein Principle of Separability in a "time-dependent" version of the atomic photon-cascade-correlation experiment done by Clauser (1972). This latter experiment gave results which upheld the QED predictions about photon states. However, since these photon states exist in both QED and QEMED, Clauser's experiment also supports the validity of QEMED. Clauser only ruled out "local hidden variable" LHV theories and "neoclassical radiation" NCT theories. These experiments have not yet ruled out QEMED, in favor of QED, in the specific context of the Einstein Principle of Separability. In Aspect's experiment, now being developed and carried out, the Einstein Principle of Separability will be tested in an atomic photon-cascade-correlation experiment, in which the measuring instrument polarizers will attempt to measure the photon correlations while rapidly changing their orientations. If the Einstein Principle of Separability is valid in Nature, then the QED photon correlation prediction should break down when the polarizer flipping time is faster than the light travel time between the two measuring polarizers, since they will be separated by spacelike points in space-time. If this occurs, then QED is wrong and QEMED is correct! On the other hand, if the Einstein Principle of Separability is not observed in Aspect's experiment, then QED is correct and QEMED is wrong!

[2]The details of this will be elaborated on in a forthcoming sequel to this paper, in a future issue of this journal.

In essence QEMED is suggesting that the quantum measurement process is fully microscopic, with the "object to be measured" and the "measuring instrument" belonging to the same level of microscopic functioning in the QEMED operator equations of motion, and associated Hilbert space.[3] This differs from QED, which carries with it the Bohr (1958, 1963) axiom that "the objects measured in a quantum measurement process have a different status in the theory to the measurement instruments, in that they belong to different levels of functioning which are not related mechanically...." Hence the forthcoming test of the Einstein Principle of Separability, by Aspect, will also be testing the validity of QEMED, and its "fully microscopic measurement paradigm," against QED and its inherent "Bohr-complimentarity measurement paradigm." It is of fundamental importance to see what future calculations and experiments have to say about operationally distinguishing between the predictions of QED, and the new QEMED formulation of electron–positron–photon processes in the microcosm, since QEMED represents a quantum field theory which contains a proper union of the special theory of relativity and the quantum measurement paradigm at the microscopic level.

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[3]Hence QEMED does not invoke the Copenhagen Interpretation of the square of the amplitudes, as on *exact* prescription for measurement. Rather QEMED is compatible with Wheeler's (1979) axiom that "No elementary phenomenon is a physical phenomenon until it is a recorded phenomenon."

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